

# Strongly correlated two-photon transport in a one-dimensional waveguide coupled to a weakly nonlinear cavity

Xun-Wei Xu<sup>1</sup> and Yong Li<sup>1,2,\*</sup>

<sup>1</sup>Beijing Computational Science Research Center, Beijing 100084, China

<sup>2</sup>Synergetic Innovation Center of Quantum Information and Quantum Physics,  
University of Science and Technology of China, Hefei, Anhui 230026, China

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We study the photon-photon correlation properties of two-photon transport in a one-dimensional waveguide coupled to a nonlinear cavity via a real-space approach. It is shown that the intrinsic dissipation of the nonlinear cavity has an important effect upon the correlation of the transported photons. More importantly, strongly correlated photons can be obtained in the transmitted photons even when the nonlinear interaction strength is weak in the cavity. The strong photon-photon correlation is induced by the Fano resonance involving destructive interference between the plane wave and bound state for two-photon transport.

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## I. INTRODUCTION

Strong photon-photon interaction is one of the fundamental conditions for the application of single photons in quantum information processing. Imamoglu *et al.* proposed to create strong photon-photon interaction by a high-finesse cavity containing a low-density four-level atomic medium [1]. They theoretically showed that strong antibunching effect can be observed in the transmitted photons when the Kerr nonlinearity induced by the resonant atomic medium is strong. The underlying physics can be understood as the excitation of a first photon blocking the transport of a second photon for the strong nonlinearity in the cavity, which is referred as the photon blockade effect [1]. Weakly driven cavity with strong nonlinearity is one of the systems for creating strongly interacting photons. Beyond the traditional nonlinear optics, strong interactions between single atoms and photons have already been demonstrated in the cavity quantum electrodynamics systems [2–5], and photon blockade has been observed in the regime for strong atom-cavity coupling [6–10].

Shen and Fan proposed another scheme to create strong photon-photon interaction [11, 12]. They showed that two-photon transport is strongly correlated in one-dimensional (1D) waveguide coupled to a two-level system [11, 12]. This strong correlation arises from the interference between the reemitted and scattered waves in the 1D waveguide. Photon-photon bound states appear due to the strong photon-photon interaction and induce the photon blockade effect [13–15]. Moreover, Roy proposed to realize an optical diode at few-photon level by two- or multi-photon transport in a 1D waveguide coupled asymmetrically to a two-level system [16], or to generate a probe to detect atomic level structures by two-photon scattering in a 1D waveguide coupled to different atomic system with similar transition energies [17–19]. Strong photon-photon interaction can also be created by two-photon transport in a 1D waveguide coupled to a cavity con-

taining a two-level atom [20, 21], two two-level atoms coupled via the Rydberg interaction [22], a Kerr medium [23], or a four-level atom [24].

Recently, Liao and Law [23] investigated the transport properties of two photons inside a 1D waveguide side-coupled to a single-mode nonlinear cavity based on the Laplace transform method. They treated the nonlinear cavity as a perfect one with no intrinsic dissipation, which is only applicable to the condition that the intrinsic dissipation is much smaller than the coupling between the 1D waveguide and single-mode cavity. In Refs. [26, 27], it has been revealed that the dissipation of the cavity has a distinct effect on the single-photon transmission properties. To date, there exists little literature on the subject that how the intrinsic dissipation of cavity affects the transport properties of the two photons inside a 1D waveguide side-coupled to a single-mode cavity with nonlinear medium. What's more, Liao and Law have shown that strong photon-photon correlation between two transmitted or reflected photons is based on strong nonlinearity [23]. However, it is difficult to obtain giant Kerr nonlinearity and low loss simultaneously. So how to create strong photon-photon correlation with weak nonlinearity should be an interesting subject.

In this paper, we will study the photon-photon correlation as two-photon transport in 1D waveguide coupled to a nonlinear cavity by the real-space approach [25–27] with taking the effect of the intrinsic loss of the nonlinear cavity into account. It is shown that the correlation of the transport photons is significantly dependent on the intrinsic dissipation of the nonlinear cavity when the intrinsic dissipation is of the order of the coupling between the 1D waveguide and nonlinear cavity mode. More importantly, we find that the strong photon-photon correlation can be created even with weak nonlinearity in the cavity.

The paper is organized as follows: In Sec. II, we show the physical model for the transport of two photons inside a 1D waveguide coupled to a nonlinear cavity. The photon-photon correlation properties for the cavity with strong and weak nonlinearity are investigated in Sec. III and IV, respectively. Finally, we draw our conclusions in Sec. V.

\*Electronic address: liyong@csrc.ac.cn

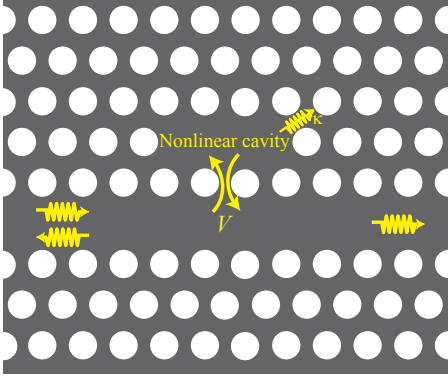


FIG. 1: (Color online) Schematic diagram of a 1D waveguide coupled to a cavity with Kerr-type nonlinear medium. Photons injected into the waveguide from the left side are scattered by the nonlinear cavity, so the photons are reflected or transmitted in the waveguide.

## II. PHYSICAL MODEL

As shown in Fig. 1, the system consists of a 1D waveguide (a row defect waveguide) evanescently coupled to a nonlinear photonic crystal cavity with coupling constant  $V$  [28–30]. By incorporating the excitation amplitudes of the reservoir and in a frame rotating at a frequency far away from the cutoff frequency of the dispersion for linearizing [26, 27], the effective Hamiltonian of the system is [25] ( $\hbar = 1$ ):

$$\begin{aligned}
 H = & -iv_c \int dx \left[ c_R^\dagger(x) \frac{\partial}{\partial x} c_R(x) - c_L^\dagger(x) \frac{\partial}{\partial x} c_L(x) \right] \\
 & + \int dx V \delta(x) \left[ c_R^\dagger(x) a + c_L^\dagger(x) a + \text{H.c.} \right] \\
 & + \left( \omega_a - i\frac{\kappa}{2} \right) a^\dagger a + U a^\dagger a^\dagger a a, \quad (1)
 \end{aligned}$$

where  $c_R^\dagger(x)$  [ $c_L^\dagger(x)$ ] is the bosonic operator creating a right-going (left-going) photon with group velocity  $v_c$  at position  $x$ ;  $a$  ( $a^\dagger$ ) is the annihilation (creation) operator of the cavity mode with frequency  $\omega_a$ ; the cavity is filled with Kerr medium with nonlinear interaction strength  $U$  and intrinsic dissipation rate  $\kappa$ . By employing  $c_e^\dagger(x) = [c_R^\dagger(x) + c_L^\dagger(-x)]/\sqrt{2}$ ,  $c_o^\dagger(x) = [c_R^\dagger(x) - c_L^\dagger(-x)]/\sqrt{2}$ , the Hamiltonian is transformed into:

$$\begin{aligned}
 H = & -iv_c \int dx \left[ c_e^\dagger(x) \frac{\partial}{\partial x} c_e(x) + c_o^\dagger(x) \frac{\partial}{\partial x} c_o(x) \right] \\
 & + \int dx \bar{V} \delta(x) \left[ c_e^\dagger(x) a + c_o^\dagger(x) a^\dagger \right] \\
 & + \left( \omega_a - i\frac{\kappa}{2} \right) a^\dagger a + U a^\dagger a^\dagger a a. \quad (2)
 \end{aligned}$$

Here, the right- and left-going modes in the waveguide are transformed to the even and odd modes  $\{c_e^\dagger(x), c_o^\dagger(x)\}$ , and the cavity mode only couples to the even mode with effective coupling constant  $\bar{V} = \sqrt{2}V$ .

Assume that there are two photons injected into the waveguide from the left side with momenta  $k_1$  and  $k_2$ , respectively.

The wave function for the two photons before scattering (incoming state) is given by

$$|\Psi_i\rangle = \int \int dx_1 dx_2 \phi_k(x_1, x_2) \frac{1}{\sqrt{2}} c_R^\dagger(x_1) c_R^\dagger(x_2) |\emptyset\rangle, \quad (3)$$

where  $\phi_k(x_1, x_2) = (e^{ik_1 x_1 + ik_2 x_2} + e^{ik_1 x_2 + ik_2 x_1})/(2\sqrt{2}\pi)$ ,  $|\emptyset\rangle$  is the vacuum state of the system. The general two-photon scattering state for two incident photons in the right-going mode can be obtained by solving the Schrödinger equation with the incoming state [Eq. (3)] (see Appendix A).

The asymptotic two-photon outgoing scattering state in the spaces with the right- and left-going modes is composed of two transmitted, two reflected, and one transmitted plus one reflected photons as follows [12, 17, 18]

$$\begin{aligned}
 & \int \int dx_1 dx_2 t(x_1, x_2) \frac{1}{\sqrt{2}} c_R^\dagger(x_1) c_R^\dagger(x_2) |\emptyset, g\rangle \\
 & + \int \int dx_1 dx_2 r(x_1, x_2) \frac{1}{\sqrt{2}} c_L^\dagger(x_1) c_L^\dagger(x_2) |\emptyset, g\rangle \\
 & + \int \int dx_1 dx_2 rt(x_1, x_2) c_L^\dagger(x_1) c_R^\dagger(x_2) |\emptyset, g\rangle, \quad (4)
 \end{aligned}$$

where

$$t(x_1, x_2) = t_p(x_1, x_2) + t_b(x_1, x_2), \quad (5)$$

$$r(x_1, x_2) = r_p(x_1, x_2) + r_b(x_1, x_2), \quad (6)$$

$$rt(x_1, x_2) = rt_p(x_1, x_2) + rt_b(x_1, x_2). \quad (7)$$

$t_p(x_1, x_2)$ ,  $r_p(x_1, x_2)$  and  $rt_p(x_1, x_2)$  correspond to the plane wave part,

$$t_p(x_1, x_2) = \phi_k(x_1, x_2) \bar{t}_{k_1} \bar{t}_{k_2}, \quad (8)$$

$$r_p(x_1, x_2) = \phi_k(-x_1, -x_2) \bar{r}_{k_1} \bar{r}_{k_2}, \quad (9)$$

$$\begin{aligned}
 rt_p(x_1, x_2) = & \frac{1}{2\pi} e^{-i\frac{\omega}{2}\frac{x}{v_c}} \left( \bar{r}_{k_2} \bar{t}_{k_1} e^{i2\Delta_1 \frac{x_c}{v_c}} \right. \\
 & \left. + \bar{t}_{k_2} \bar{r}_{k_1} e^{-i2\Delta_1 \frac{x_c}{v_c}} \right), \quad (10)
 \end{aligned}$$

where

$$\bar{t}_{k_i} = \frac{v_c k_i - \omega_a + i\frac{\kappa}{2}}{v_c k_i - \omega_a + i\frac{\kappa+\Gamma}{2}}, \quad (11)$$

$$\bar{r}_{k_i} = \frac{-i\frac{\Gamma}{2}}{v_c k_i - \omega_a + i\frac{\kappa+\Gamma}{2}} \quad (12)$$

are the single-photon transmission and reflection amplitudes,  $\Gamma = \bar{V}^2/v_c$ ,  $\Delta_i = v_c k_i - \omega/2$  and  $\Delta_a = \omega_a - \omega/2$ ,  $x_c = (x_2 + x_1)/2$  and  $x = x_2 - x_1$  are the center-of-mass and the relative coordinates, respectively.  $t_b(x_1, x_2)$ ,  $r_b(x_1, x_2)$  and  $rt_b(x_1, x_2)$  are contributions from the two-photon bound state,

$$t_b(x_1, x_2) = \frac{1}{4} B e^{i\omega \frac{x_c}{v_c}} e^{[-i2\Delta_a - (\kappa+\Gamma)] \frac{|x|}{2v_c}}, \quad (13)$$

$$r_b(x_1, x_2) = \frac{1}{4} B e^{-i\omega \frac{x_c}{v_c}} e^{[-i2\Delta_a - (\kappa+\Gamma)] \frac{|x|}{2v_c}}, \quad (14)$$

$$rt_b(x_1, x_2) = \frac{1}{2\sqrt{2}} B e^{-i\frac{\omega}{2}\frac{x}{v_c}} e^{[-i2\Delta_a - (\kappa+\Gamma)] \frac{|x_c|}{v_c}}. \quad (15)$$

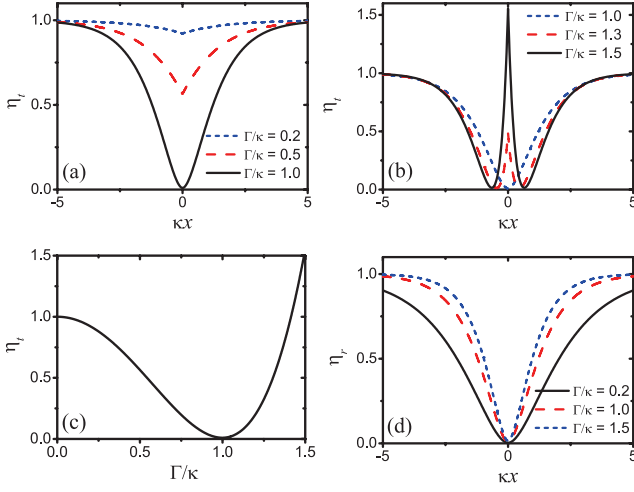


FIG. 2: (Color online) (a) and (b)  $\eta_t$  and  $\eta_r$  plotted as a function of relative coordinate  $\kappa x$  for different values of the coupling constant  $\Gamma/\kappa$ ; (c)  $\eta_t$  plotted as a function of  $\Gamma/\kappa$  for  $x = 0$ ; (d)  $\eta_r$  plotted as a function of relative coordinate  $\kappa x$  for different values of  $\Gamma/\kappa$ . The other parameters are  $\Delta_1 = \Delta_2 = \Delta_a = 0$ ,  $U/\kappa = 10$ .

where  $B$  is dependent on the nonlinear interaction strength  $U$  as given in Appendix A [Eq. (A30)].

Let us introduce two quantities to characterize the correlation between the two transmitted photons and the correlation between the two reflected photons:

$$\eta_t = \frac{|t(x_1, x_2)|^2}{|t_p(x_1, x_2)|^2}, \quad (16)$$

$$\eta_r = \frac{|r(x_1, x_2)|^2}{|r_p(x_1, x_2)|^2}. \quad (17)$$

$|t(x_1, x_2)|^2$  and  $|r(x_1, x_2)|^2$  are the probabilities for two-photon transmission and two-photon reflection, respectively;  $|t_p(x_1, x_2)|^2$  and  $|r_p(x_1, x_2)|^2$  are the probabilities of two photons being transmitted and reflected by the system without photon-photon interaction because of  $B = 0$  for  $U = 0$ . So if two photons are transmitted (reflected) independently, we have  $\eta_t = 1$  ( $\eta_r = 1$ ); if  $\eta_t \neq 1$  or  $\eta_r \neq 1$ , then we have correlated two-photon state. As  $\eta_t < 1$  ( $\eta_r < 1$ ), effective repulsive interaction between the two transmitted (reflected) photons is induced, causing suppression on two-photon transmission (reflection), then photon blockade emerges; on the contrary, as  $\eta_t > 1$  ( $\eta_r > 1$ ), there is effective attractive interaction between the two transmitted (reflected) photons, causing enhanced two-photon transmission (reflection), and photon-induced tunneling appears.

### III. STRONGLY NONLINEAR REGIME

In this section, we will study the effect of the intrinsic dissipation of the cavity on the two-photon transport properties with a strongly Kerr nonlinear medium in the cavity ( $U > \kappa$ ). We assume that the two photons are both resonant with the

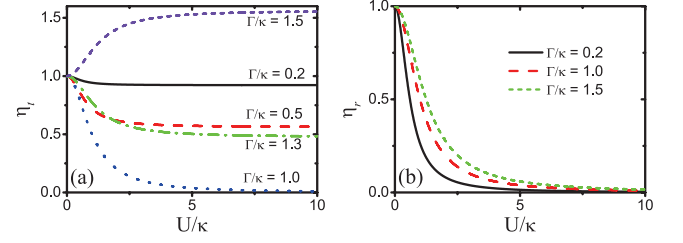


FIG. 3: (Color online) (a)  $\eta_t$  and (b)  $\eta_r$  plotted as functions of  $U/\kappa$  for different values of  $\Gamma/\kappa$  with  $x = 0$  and  $\Delta_1 = \Delta_2 = \Delta_a = 0$ .

cavity mode, i.e.  $\Delta_1 = \Delta_2 = \Delta_a = 0$ . In Figs. 2(a) and (b), we show  $\eta_t$  as a function of relative coordinate  $\kappa x$  for different values of the coupling parameter  $\Gamma/\kappa$ . At the point  $x = 0$ ,  $\eta_t$  decreases from one to nearly zero as  $\Gamma/\kappa$  increases from zero to one, then increases rapidly by the further increase of  $\Gamma/\kappa$  when  $\Gamma/\kappa > 1$ , as shown in Fig. 2(c). Similar effect was also reported in Ref. [13] for a coherent state transport in a 1D waveguide coupled to a single two-level system.  $\eta_t \approx 0$  at the point of  $\Gamma/\kappa = 1$  shows that the two transmitted photons are strongly repulsive after transmitting;  $\eta_t > 1$  in the regime of  $\Gamma/\kappa > 1.4$  implies that the two transmitted photons are attractive after transmitting. So we can make the transmitted photons from repulsive to attractive by changing the coupling parameter  $\Gamma/\kappa$ .  $\eta_r$  as a function of relative coordinate  $\kappa x$  for different values of the coupling parameter  $\Gamma/\kappa$  is shown in Fig. 2(d). Around the point  $x = 0$ ,  $\eta_r \approx 0$ , the reflected photons are strongly repulsive and the window for  $\eta_r < 0$  becomes narrower with the increase of  $\Gamma/\kappa$ .

In Fig. 3, we show  $\eta_t$  and  $\eta_r$  as functions of the nonlinear interaction strength  $U/\kappa$  for different values of the coupling parameter  $\Gamma/\kappa$ . For the transmitted photons,  $\eta_t$  increases or decreases monotonously with the increase of  $U$  depending on the value of  $\Gamma/\kappa$  and reaches the saturation point when  $U \gg \kappa$ . If we want to obtain strongly repulsive transmitted photons, we should set  $\Gamma/\kappa = 1$ ; on the contrary, if we need strongly attractive transmitted photons, we can set  $\Gamma/\kappa \gg 1$ . For the reflected photons,  $\eta_r$  decreases monotonously as the increase of  $U$  and is close to zero when  $U \gg \kappa$  and decreases more slowly with the increase of  $\Gamma/\kappa$ . That is to say the reflected photons represent strongly repulsive interaction in the strongly nonlinear interaction condition ( $U \gg \kappa, \Gamma$ ) for  $\Delta_1 = \Delta_2 = \Delta_a = 0$ .

### IV. WEAKLY NONLINEAR REGIME

Next, we will investigate whether strong photon-photon correlation can be created with weak nonlinearity in the cavity ( $U < \kappa$ ).  $\eta_t$  and  $\eta_r$  are plotted as functions of the detuning  $\Delta_a/\kappa$  in Fig. 4(a) for  $\Delta_1 = \Delta_2 = 0$ ,  $\Gamma = 100\kappa$ , and  $U = \kappa/100$ . We can see that there is an optimal point for the transmitted photons exhibiting strongly repulsive interaction ( $\eta_t \approx 0$ ) at  $\Delta_a = \kappa/2$ , while  $\eta_r \approx 1$ . The result shows that as the coupling between the cavity mode and 1D waveguide is strong ( $\Gamma = 100\kappa$ ), the transmitted photons can exhibit

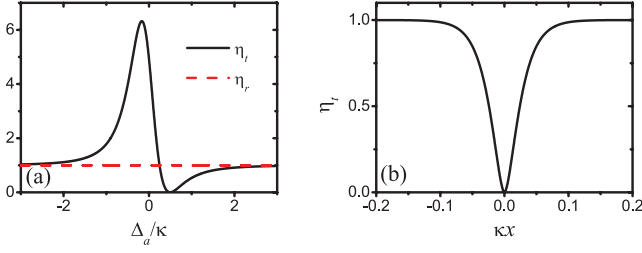


FIG. 4: (Color online) (a)  $\eta_t$  and  $\eta_r$  plotted as functions of the detuning  $\Delta_a/\kappa$  for  $x = 0$ , (b)  $\eta_t$  as a function of relative coordinate  $\kappa x$  for  $\Delta_a = \kappa/2$ . The other parameters are  $\Gamma = 100\kappa$ ,  $U = \kappa/100$  and  $\Delta_1 = \Delta_2 = 0$ .

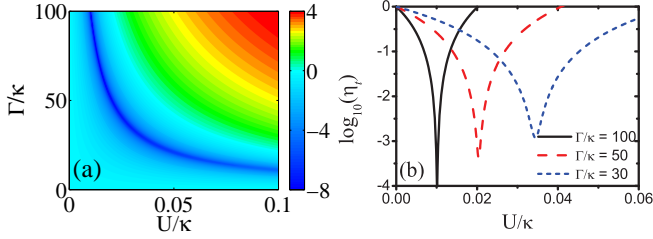


FIG. 5: (Color online) (a)  $\log_{10}(\eta_t)$  plotted as a function of  $U/\kappa$  and  $\Gamma/\kappa$ . (b)  $\log_{10}(\eta_t)$  plotted as a function of  $U/\kappa$  for different values of  $\Gamma/\kappa$ . The detunings  $\Delta_a = \kappa/2$ ,  $\Delta_1 = \Delta_2 = 0$ , and  $x = 0$ .

strongly repulsive interaction even when the nonlinear interaction in the cavity is weak ( $U = \kappa/100$ ).  $\eta_t$  as a function of relative coordinate  $\kappa x$  is shown in Fig. 4(b). The spacescale of the repulsive interaction is much smaller than that of the cases in Figs. 2(b) and (d).

To figure out the optimal conditions for obtaining the strongly repulsive photon-photon interaction in the weakly nonlinear condition, a two-dimensional plot of  $\log_{10}(\eta_t)$  as a function of the nonlinear interaction strength  $U/\kappa$  and coupling parameter  $\Gamma/\kappa$  for  $\Delta_a = \kappa/2$  is shown in Fig. 5(a). With increasing  $\Gamma/\kappa$ , the value of  $U/\kappa$  for getting the strongly repulsive interaction (dark blue regime in the figure) descends gradually. From the cuts of the color scale plot shown in Fig. 5(b), for  $\Delta_a = \kappa/2$ , the optimal strongly repulsive interaction occurs along the curve  $\Gamma/\kappa \approx \kappa/U$ .

In order to understand the origin of the strongly repulsive interaction, we will derive the optimal conditions analytically after doing some approximations. From Eqs. (5) and (8), as  $x_1 = x_2 = x_c$  and  $\Delta_1 = \Delta_2 = 0$ , we have

$$t(x_1, x_2) = \frac{\sqrt{2}}{2\pi} e^{i\omega \frac{x_c}{v_c}} \bar{t}_{k_1} \bar{t}_{k_2} \chi, \quad (18)$$

$$t_p(x_1, x_2) = \frac{\sqrt{2}}{2\pi} e^{i\omega \frac{x_c}{v_c}} \bar{t}_{k_1} \bar{t}_{k_2} \quad (19)$$

with

$$\chi = 1 + \left(\frac{\Gamma}{2}\right)^2 \frac{U}{(\Delta_a + U - i\frac{\kappa + \Gamma}{2})(\Delta_a - i\frac{\kappa}{2})^2}. \quad (20)$$

In the condition that  $\Gamma \gg \Delta_a \sim \kappa \gg U$ ,  $\eta_t$  can be written in

a Fano-like line shape approximately,

$$\eta_t \approx \frac{\epsilon^2 + (1+q)^2}{1 + \epsilon^2} \quad (21)$$

with the parameters

$$q = \frac{U\Gamma^2}{2\Delta_a(2\Delta_a^2 - \kappa\Gamma)}, \quad (22)$$

$$\epsilon = \frac{\Gamma(4\Delta_a^2 - \kappa^2)}{4\Delta_a(2\Delta_a^2 - \kappa\Gamma)}. \quad (23)$$

Eq. (21) suggests that there is one minimum in the Fano-like profile at

$$q = 1, \quad \epsilon = 0, \quad (24)$$

resulting in the optimal conditions for strongly repulsive interaction,

$$\Delta_a = \frac{\kappa}{2}, \quad \frac{U}{\kappa} = \frac{\kappa}{\Gamma}. \quad (25)$$

After substituting the optimal conditions [Eq. (25)] back into Eq. (16), we have

$$\eta_t \approx \left(\frac{U}{\kappa}\right)^2. \quad (26)$$

These analytical expressions [Eqs. (25) and (26)] agree well with the numerical results shown in Fig. 5.

The strongly repulsive photon-photon interaction for the transmitted photons comes from the destructive interference effect between different paths for two-photon transmitting in the coupling system with Fano resonances [31]. The two photons can pass independently through the 1D waveguide coupled to a nonlinear cavity as plan waves given by  $t_p(x_1, x_2)$  or instead pass through the system together with a bound state given by  $t_b(x_1, x_2)$ , which is dependent on the strength of the nonlinearity. As  $\Gamma \gg \kappa$ , the two photons are mainly resonantly reflected by the cavity in the regime  $|\Delta_a| < V$ , i.e.  $|r_p(x_1, x_2)|^2 \approx 1$  and  $|t_p(x_1, x_2)|^2 \approx 0$ , so that the nonlinear interaction strength  $U$  [in  $t_b(x_1, x_2)$ ] required for quantum destructive interference can be much smaller than the intrinsic dissipation rate  $\kappa$ .

## V. CONCLUSIONS

In conclusion, we have studied the transport properties of two photons in a 1D waveguide evanescently coupled to a nonlinear cavity. We have shown that the correlation of the transport photons is strongly dependent on the intrinsic dissipation of the nonlinear cavity, and this provides us an effective way to control the correlation properties for the transmitted two photons. What's more, due to the Fano resonances involving destructive interference effect between the plane wave and bound state for two-photon transmission, the transmitted two photons exhibit strongly repulsive interaction in the optimal conditions even with weak nonlinearity in the cavity.

It is worth noting that the strong photon antibunching in the presence of weak nonlinearity has also been found in two coupled nonlinear cavities (nonlinear photonic molecule) theoretically [32–35]. The main difference is that the strong photon antibunching was attributed to the ordinary destructive quantum interference effect in the nonlinear photonic molecule [33]. In addition, various nonlinear optical systems are proposed to achieve strong photon blockade with weak nonlinearity, such as bimodal optical cavity with a quantum dot [36, 37], coupled optomechanical systems [38, 39], or coupled single-mode cavities with second- or third-order nonlinearity [40–42]. Similarly, if a 1D waveguide is side-coupled to a cavity with a quantum dot, a second- or third-order nonlinear medium, or a mechanical resonator [43–46] coupled to the cavity mode, the strong photon-photon interaction may also be obtained in the transmitted photons even when the effective nonlinearity in the cavity is weak.

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## Appendix A: Two-photon scattering by nonlinear cavity

In this appendix we provide a derivation of the scattering states for two incident photons in the right-going mode. Decomposing the right-going mode to the even and odd modes by  $c_R^\dagger(x) = [c_e^\dagger(x) + c_o^\dagger(x)]/\sqrt{2}$ , the incoming state [Eq. (3)] can be rewritten as

$$\begin{aligned} |\Psi_i\rangle = & \frac{1}{2} \int \int dx_1 dx_2 \frac{1}{2\pi\sqrt{2}} (e^{ik_1 x_1 + ik_2 x_2} + e^{ik_1 x_2 + ik_2 x_1}) \frac{1}{\sqrt{2}} c_e^\dagger(x_1) c_e^\dagger(x_2) |\emptyset\rangle \\ & + \frac{1}{2} \int \int dx_1 dx_2 \frac{1}{2\pi\sqrt{2}} (e^{ik_1 x_1 + ik_2 x_2} + e^{ik_1 x_2 + ik_2 x_1}) \frac{1}{\sqrt{2}} c_o^\dagger(x_1) c_o^\dagger(x_2) |\emptyset\rangle \\ & + \frac{1}{\sqrt{2}} \int \int dx_1 dx_2 \frac{1}{2\pi\sqrt{2}} (e^{ik_1 x_1 + ik_2 x_2} + e^{ik_1 x_2 + ik_2 x_1}) c_e^\dagger(x_1) c_o^\dagger(x_2) |\emptyset\rangle. \end{aligned} \quad (\text{A1})$$

We will determine the scattering state in the spaces spanned by the even and odd modes.

The general two-photon scattering state of the system in the spaces with even and odd modes takes the following form:

$$|\Psi\rangle = \frac{1}{2} |\Psi_{ee}\rangle + \frac{1}{2\sqrt{2}} |\Psi_{oe}\rangle + \frac{1}{2\sqrt{2}} |\Psi_{eo}\rangle + \frac{1}{2} |\Psi_{oo}\rangle, \quad (\text{A2})$$

with

$$\begin{aligned} |\Psi_{ee}\rangle = & \int \int dx_1 dx_2 \phi_{ee}(x_1, x_2) \frac{1}{\sqrt{2}} c_e^\dagger(x_1) c_e^\dagger(x_2) |\emptyset\rangle + \int dx \phi_{ae}(x) c_e^\dagger(x) a^\dagger |\emptyset\rangle \\ & + \phi_{aa} \frac{1}{\sqrt{2}} a^\dagger a^\dagger |\emptyset\rangle, \end{aligned} \quad (\text{A3})$$

$$|\Psi_{oe}\rangle = \int \int dx_1 dx_2 \phi_{oe}(x_1, x_2) c_o^\dagger(x_1) c_e^\dagger(x_2) |\emptyset\rangle + \int dx \phi_{oa}(x) c_o^\dagger(x) a^\dagger |\emptyset\rangle, \quad (\text{A4})$$

$$|\Psi_{eo}\rangle = \int \int dx_1 dx_2 \phi_{eo}(x_1, x_2) c_e^\dagger(x_1) c_o^\dagger(x_2) |\emptyset\rangle + \int dx \phi_{ao}(x) c_e^\dagger(x) a^\dagger |\emptyset\rangle, \quad (\text{A5})$$

$$|\Psi_{oo}\rangle = \int \int dx_1 dx_2 \phi_{oo}(x_1, x_2) \frac{1}{\sqrt{2}} c_o^\dagger(x_1) c_o^\dagger(x_2) |\emptyset\rangle, \quad (\text{A6})$$

where  $\phi_{ij}$  ( $i, j = e, o, a$ ) is the amplitude of the two photons with one photon in mode  $i$  and the other in mode  $j$ ; subscript  $e$  ( $o$ ) stands for even (odd) mode and subscript  $a$  stands for cavity mode. In order to satisfy the statistical property of photons, the amplitudes satisfy the relations:  $\phi_{ee}(x_1, x_2) = \phi_{ee}(x_2, x_1)$ ,  $\phi_{oo}(x_1, x_2) = \phi_{oo}(x_2, x_1)$ ,  $\phi_{eo}(x_1, x_2) = \phi_{oe}(x_2, x_1)$ , and  $\phi_{oa}(x) = \phi_{ao}(x)$ .

In this paper, we concentrate on the two-photon transport of the frequency  $\omega = v_c k_1 + v_c k_2$ . In steady state, according to the time-independent Schrödinger's equation,  $H |\Psi\rangle = \omega |\Psi\rangle$ , we obtain the following linear equations for the amplitudes in the



scattering state:

$$\left(-iv_c \frac{\partial}{\partial x_1} - iv_c \frac{\partial}{\partial x_2} - \omega\right) \phi_{ee}(x_1, x_2) + \frac{\bar{V}}{\sqrt{2}} [\delta(x_1) \phi_{ae}(x_2) + \delta(x_2) \phi_{ae}(x_1)] = 0, \quad (\text{A7})$$

$$\left(-iv_c \frac{\partial}{\partial x} + \omega_a - \omega - i\frac{\kappa}{2}\right) \phi_{ae}(x) + \sqrt{2}\bar{V}\delta(x) \phi_{aa} + \frac{\bar{V}}{\sqrt{2}} [\phi_{ee}(0, x) + \phi_{ee}(x, 0)] = 0, \quad (\text{A8})$$

$$(2\omega_a - \omega + 2U - i\kappa) \phi_{aa} + \sqrt{2}\bar{V}\phi_{ae}(0) = 0, \quad (\text{A9})$$

$$\left(-iv_c \frac{\partial}{\partial x_1} - iv_c \frac{\partial}{\partial x_2} - \omega\right) \phi_{oe}(x_1, x_2) + \bar{V}\delta(x_2) \phi_{oa}(x_1) = 0, \quad (\text{A10})$$

$$\left(-iv_c \frac{\partial}{\partial x} + \omega_a - \omega - i\frac{\kappa}{2}\right) \phi_{oa}(x) + \bar{V}\phi_{oe}(x, 0) = 0, \quad (\text{A11})$$

$$\left(-iv_c \frac{\partial}{\partial x_1} - iv_c \frac{\partial}{\partial x_2} - \omega\right) \phi_{oo}(x_1, x_2) = 0. \quad (\text{A12})$$

We use  $\phi_{ee}(x, 0) = [\phi_{ee}(x, 0^+) + \phi_{ee}(x, 0^-)]/2$ ,  $\phi_{ae}(0) = [\phi_{ae}(0^+) + \phi_{ae}(0^-)]/2$ ,  $\phi_{oe}(x, 0) = [\phi_{oe}(x, 0^+) + \phi_{oe}(x, 0^-)]/2$  for the discontinuous points. Solving the linear equations with the initial conditions [incoming state given in Eq. (A1)] and the discontinuity relations,

$$\phi_{ee}(0^+, x) - \phi_{ee}(0^-, x) = \frac{\bar{V}}{iv_c \sqrt{2}} \phi_{ae}(x), \quad (\text{A13})$$

$$\phi_{ee}(x, 0^+) - \phi_{ee}(x, 0^-) = \frac{\bar{V}}{iv_c \sqrt{2}} \phi_{ae}(x), \quad (\text{A14})$$

$$\phi_{ae}(0^+) - \phi_{ae}(0^-) = \frac{\sqrt{2}\bar{V}}{iv_c} \phi_{aa}, \quad (\text{A15})$$

$$\phi_{oe}(x_1, 0^+) - \phi_{oe}(x_1, 0^-) = \frac{\bar{V}}{iv_c} \phi_{oa}(x_1), \quad (\text{A16})$$

$$\phi_{oe}(0^+, x_2) = \phi_{oe}(0^-, x_2), \quad (\text{A17})$$

$$\phi_{oa}(0^+) = \phi_{oa}(0^-), \quad (\text{A18})$$

we find the amplitudes for the two-photon scattering state,

$$\begin{aligned} \phi_{ee}(x_1, x_2) &= \frac{1}{\sqrt{2}} [\phi_{e,k_1}(x_1) \phi_{e,k_2}(x_2) + \phi_{e,k_1}(x_2) \phi_{e,k_2}(x_1)] \\ &\quad + \left[ \theta(x_2 - x_1) \theta(x_1) B e^{i\omega \frac{x_c}{v_c}} e^{[i(\omega - 2\omega_a) - (\kappa + \Gamma)] \frac{x}{2v_c}} + (x_2 \leftrightarrow x_1) \right], \end{aligned} \quad (\text{A19})$$

$$\phi_{oe}(x_1, x_2) = \frac{1}{\sqrt{2}} [\phi_{o,k_1}(x_1) \phi_{e,k_2}(x_2) + \phi_{e,k_1}(x_2) \phi_{o,k_2}(x_1)], \quad (\text{A20})$$

$$\phi_{oo}(x_1, x_2) = \frac{1}{\sqrt{2}} [\phi_{o,k_1}(x_1) \phi_{o,k_2}(x_2) + \phi_{o,k_1}(x_2) \phi_{o,k_2}(x_1)], \quad (\text{A21})$$

$$\phi_{ae}(x_i) = \theta(-x_i) (\mu_{k_1} e^{ik_1 x_i} + \mu_{k_2} e^{ik_2 x_i}) + \theta(x_i) (\eta_{k_1} e^{ik_1 x_i} + \eta_{k_2} e^{ik_2 x_i} + \xi e^{\lambda - x_i}), \quad (\text{A22})$$

$$\phi_{oa}(x_i) = \rho_{k_1} e^{ik_1 x_i} + \rho_{k_2} e^{ik_2 x_i}, \quad (\text{A23})$$

$$\phi_{aa} = -\frac{\bar{V}}{\sqrt{2}} \frac{(\mu_{k_1} + \mu_{k_2})}{(\Delta_a + U - i\frac{\kappa + \Gamma}{2})}, \quad (\text{A24})$$

with

$$\phi_{e,k_i}(x_j) = \frac{1}{\sqrt{2\pi}} [\theta(-x_j) + t_{k_i} \theta(x_j)] e^{ik_i x_j}, \quad (\text{A25})$$

$$\phi_{o,k_i}(x_j) = \frac{1}{\sqrt{2\pi}} e^{ik_i x_j}, \quad (\text{A26})$$

where  $\theta(x)$  is the step function;  $\Gamma = \bar{V}^2/v_c$ ,  $x_c = (x_2 + x_1)/2$  and  $x = x_2 - x_1$  are the center-of-mass and the relative

coordinate, respectively;  $\Delta_i = v_c k_i - \omega/2$  and  $\Delta_a = \omega_a - \omega/2$ ;

$$t_{k_i} = \frac{\Delta_i - \Delta_a + i\frac{\kappa-\Gamma}{2}}{\Delta_i - \Delta_a + i\frac{\kappa+\Gamma}{2}}, \quad (\text{A27})$$

$$\mu_{k_1} = \frac{1}{2\pi} \frac{\bar{V}}{\Delta_2 - \Delta_a + i\frac{\kappa+\Gamma}{2}}, \quad (\text{A28})$$

$$\mu_{k_2} = \frac{1}{2\pi} \frac{\bar{V}}{\Delta_1 - \Delta_a + i\frac{\kappa+\Gamma}{2}}, \quad (\text{A29})$$

$$B = \frac{\bar{V}}{iv_c\sqrt{2}}\xi, \quad (\text{A30})$$

$$\xi = \frac{i4\pi\bar{V}U}{v_c(\Delta_a + U - i\frac{\kappa+\Gamma}{2})}\mu_{k_1}\mu_{k_2}, \quad (\text{A31})$$

and  $\eta_{k_i} = \mu_{k_i} t_{k_i}$ ,  $\rho_{k_i} = \mu_{k_i}/\sqrt{2}$ .

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